Image Reconstruction – Parallel Imaging Part I

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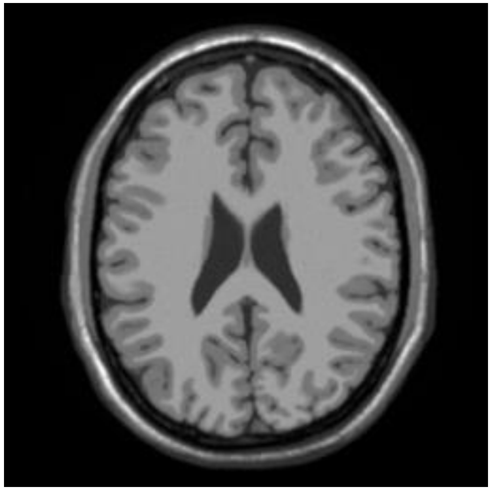
No conflicts of interest to disclose

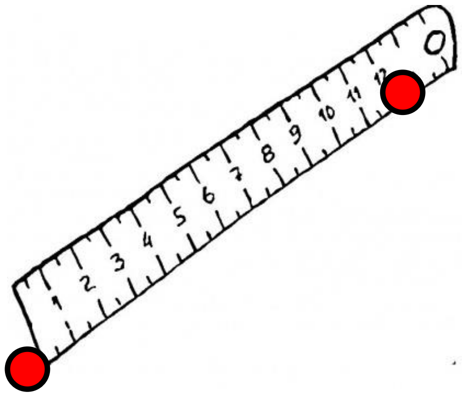
Image Reconstruction Goal

- Instantaneous results
- Perfect signal fidelity with no artifacts
- No noise

```
load im1.mat
```



How far away are we?



You are here

Goal

Efficiency
- Computation operations
- Computation time
- Memory usage

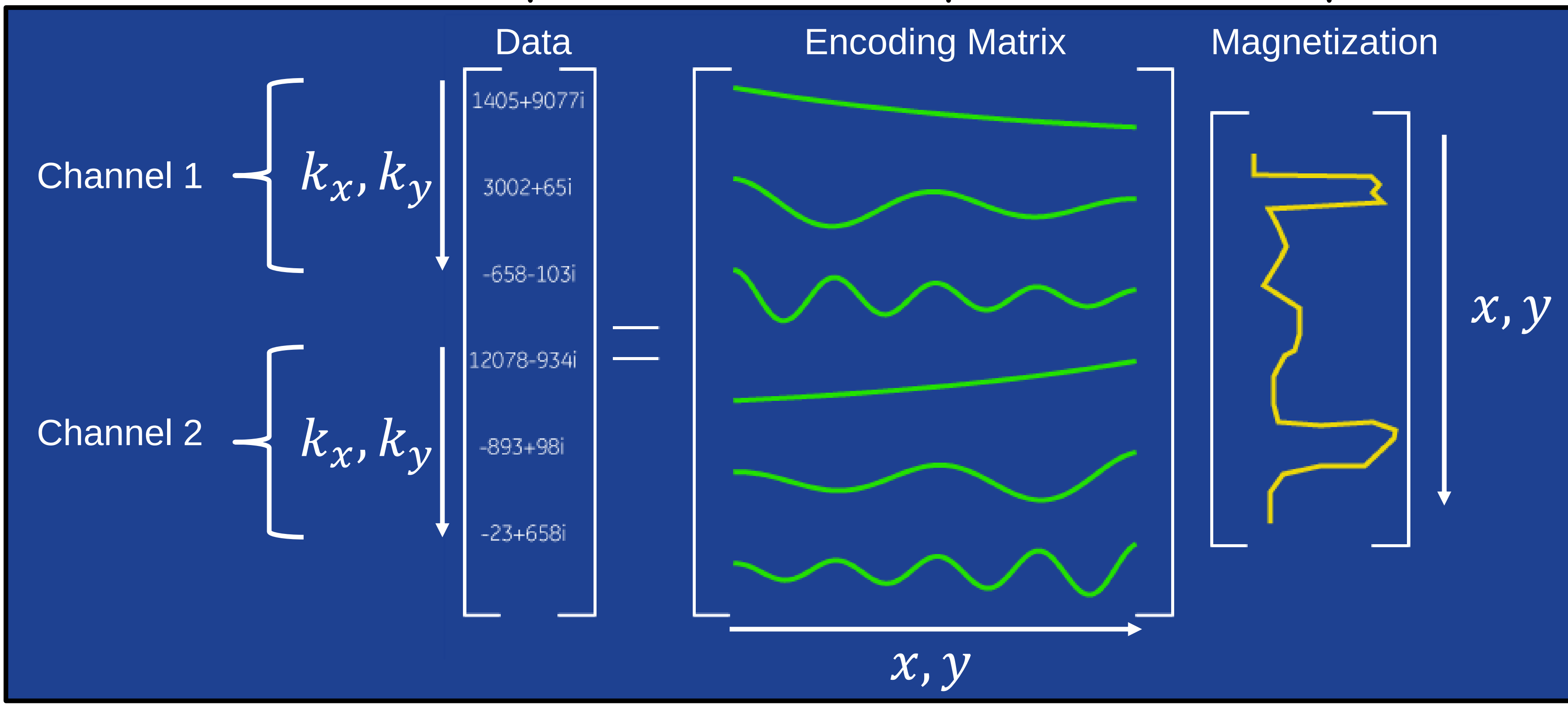
Artifacts
- Image shading
- Aliasing energy

Noise
- G-factor maps
- Noise amplification maps
- SNR-scaled reconstruction

Concept of Sensitivity Encoding

Encoding Model

$$d_j(k_x,\ k_y) = \iiint s_j(x,y)e^{i2\pi(k_x x + k_y y)} m(x,y)\, dxdy$$

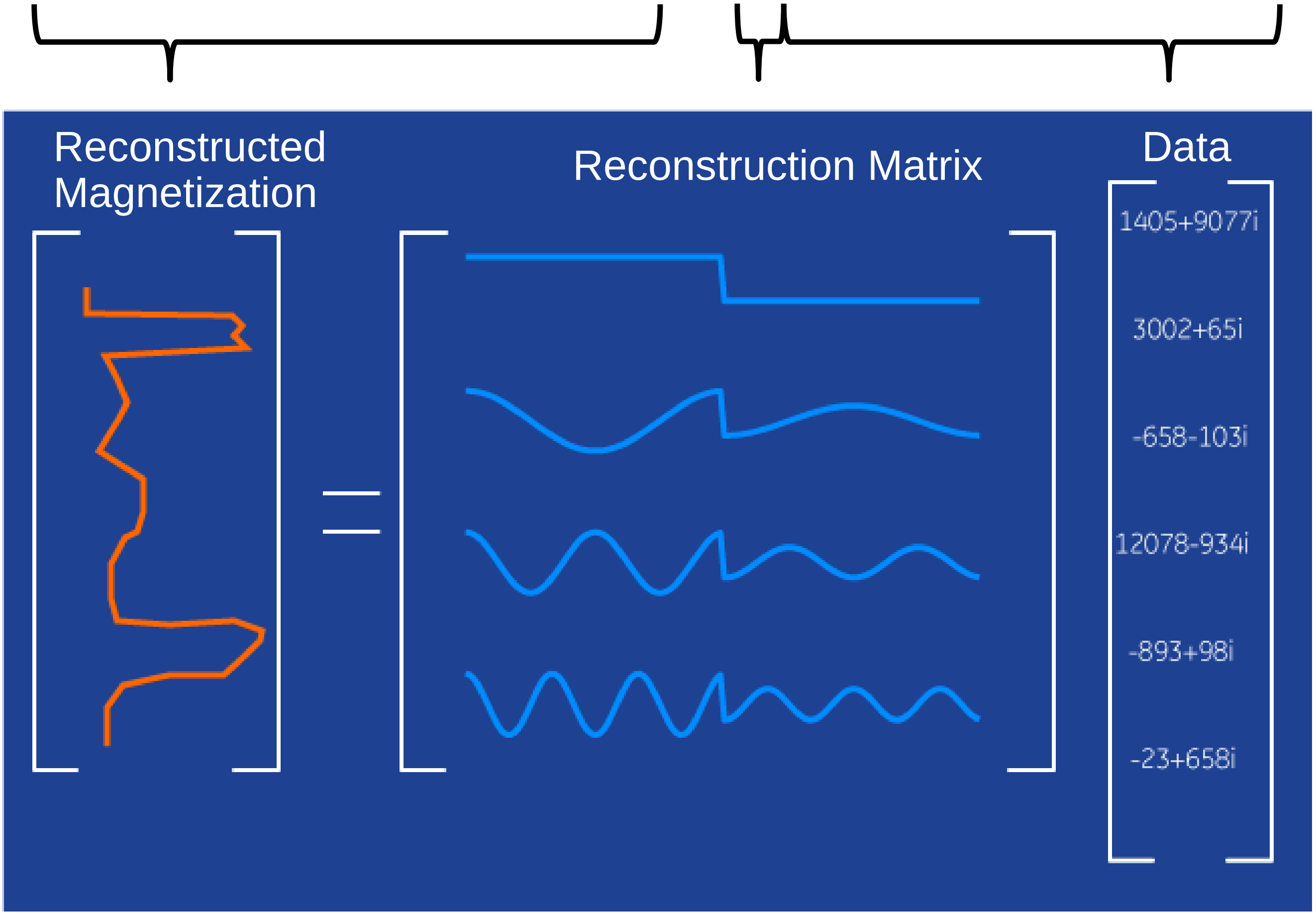


$$d = Em + \text{noise}$$

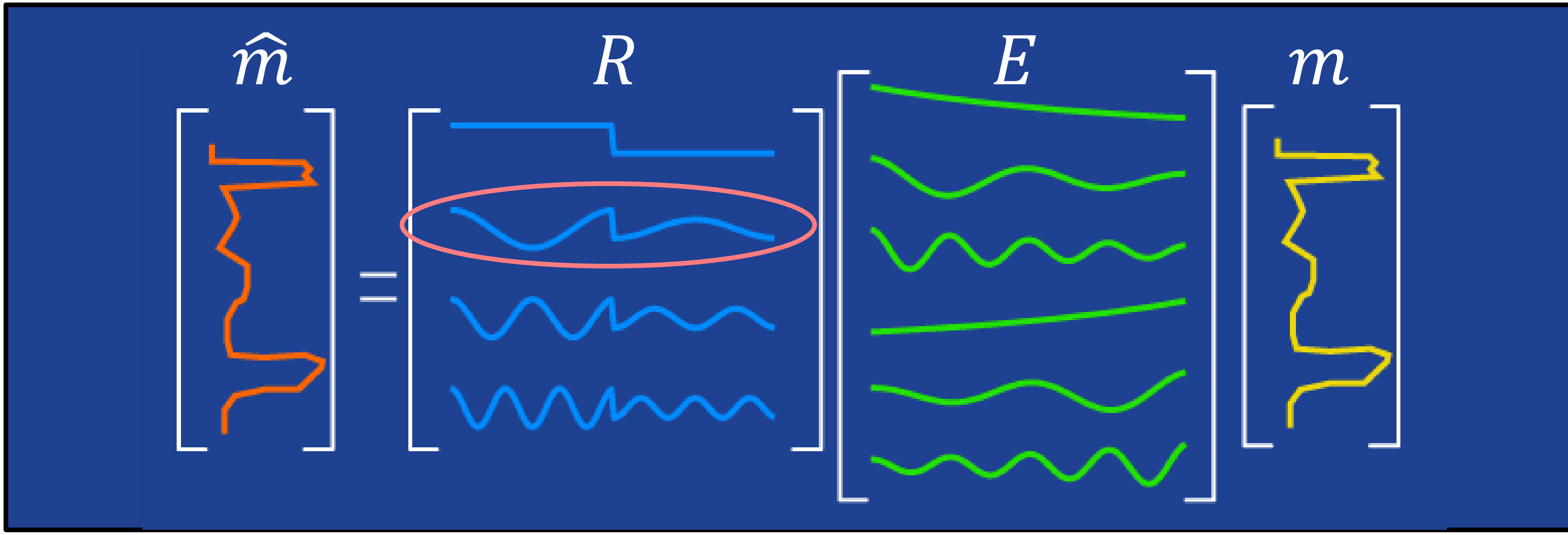
Reconstruction

$$\hat{m} = (E^H E)^{-1} E^H d$$

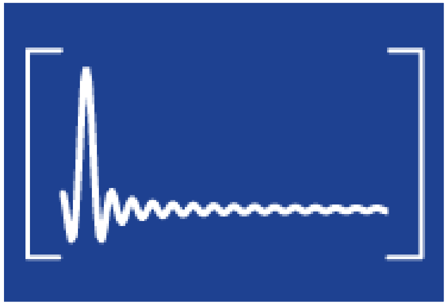
$$\hat{m} = Rd$$



Reconstruction Matrix Design



Linear combination of acquired encoding functions to give desired encoding function



Reconstruction Components

1. Estimate E
   - $s_j(x, y)$ Estimate coil sensitivities
2. Generate R
   - One possibility: $\mathrm{R} = \mathrm{pinv}(E)$
3. Apply R
   - $\hat{m} = Rd$

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Image Quality (steps 1–2)

Reconstruction Efficiency (step 3)

Reconstruction Components: Noniterative Methods

1. Estimate E
   - $s_j(x,y)$ Estimate coil sensitivities
2. Generate R
   - One possibility: $\mathrm{R} = \mathrm{pinv}(E)$
3. Apply R
   - $\widehat{m} = Rd$

} Calibration (steps 1–2)

Reconstruction Components: Iterative Methods

1. Estimate E
   - $s_j(x, y)$ Estimate coil sensitivities

} Calibration

2. Generate R
   - One possibility: $\mathrm{R} = \mathrm{pinv}(E)$
3. Apply R
   - $\hat{m} = Rd$

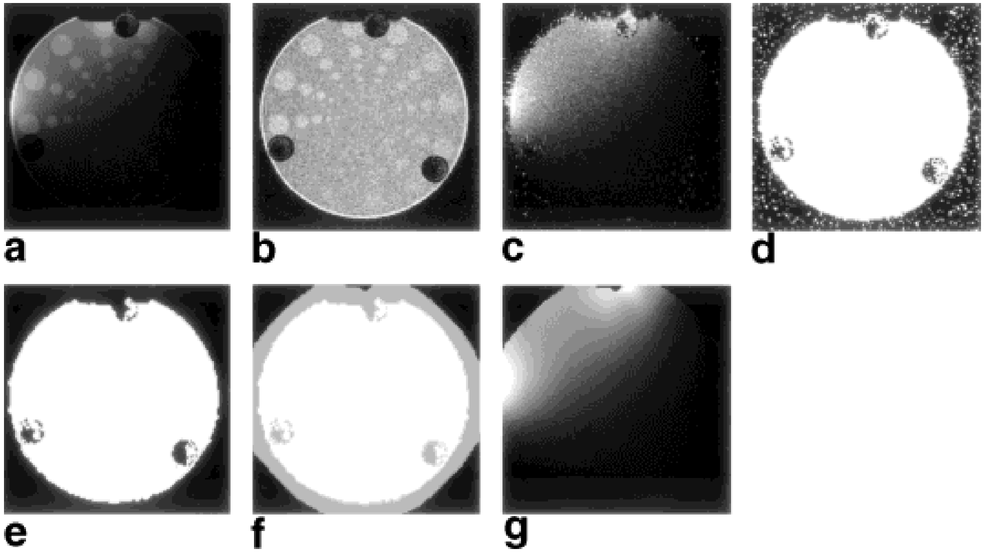
} Fused in iterative loop

Magnetic Resonance in Medicine 42:952–962 (1999)

SENSE: Sensitivity Encoding for Fast MRI

Klaas P. Pruessmann, Markus Weiger, Markus B. Scheidegger, and Peter Boesiger*

1. Estimate E
   - $s_j(x, y)$ Estimate coil sensitivities



2. Generate R
   - One possibility: $\mathrm{R} = \mathrm{pinv}(E)$

“unfolding matrix”

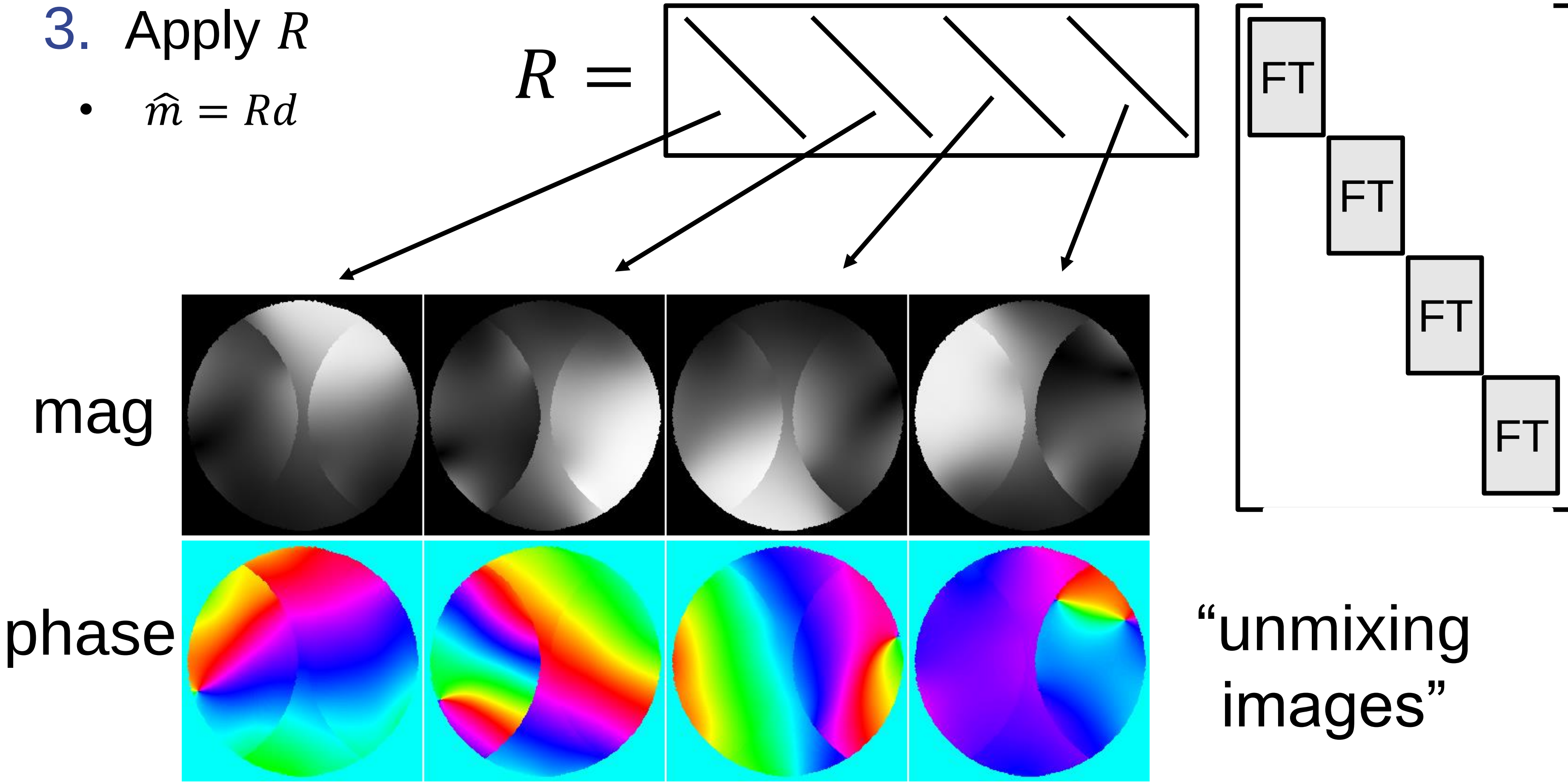
$$U = (S^H \Psi^{-1} S)^{-1} S^H \Psi^{-1},$$

Magnetic Resonance in Medicine 42:952–962 (1999)

SENSE: Sensitivity Encoding for Fast MRI

Klaas P. Pruessmann, Markus Weiger, Markus B. Scheidegger, and Peter Boesiger*

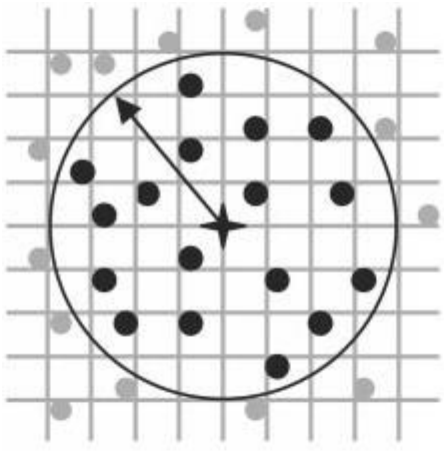
3. Apply R
   - $\hat{m} = Rd$



N_c Fourier Transforms + N_c multiplications per voxel

Local k-Space Kernels

- Enable non-iterative reconstruction of non-uniform sampling patterns.



SMASH

PARS

AUTO-SMASH

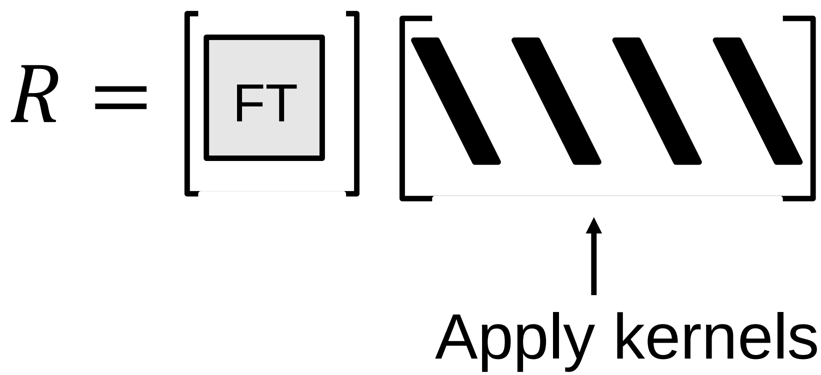
VD-AUTO-SMASH

GRAPPA

…and more

Composite Channel Local k-Space Kernels

SMASH, AUTO-SMASH, VD-AUTO-SMASH,…

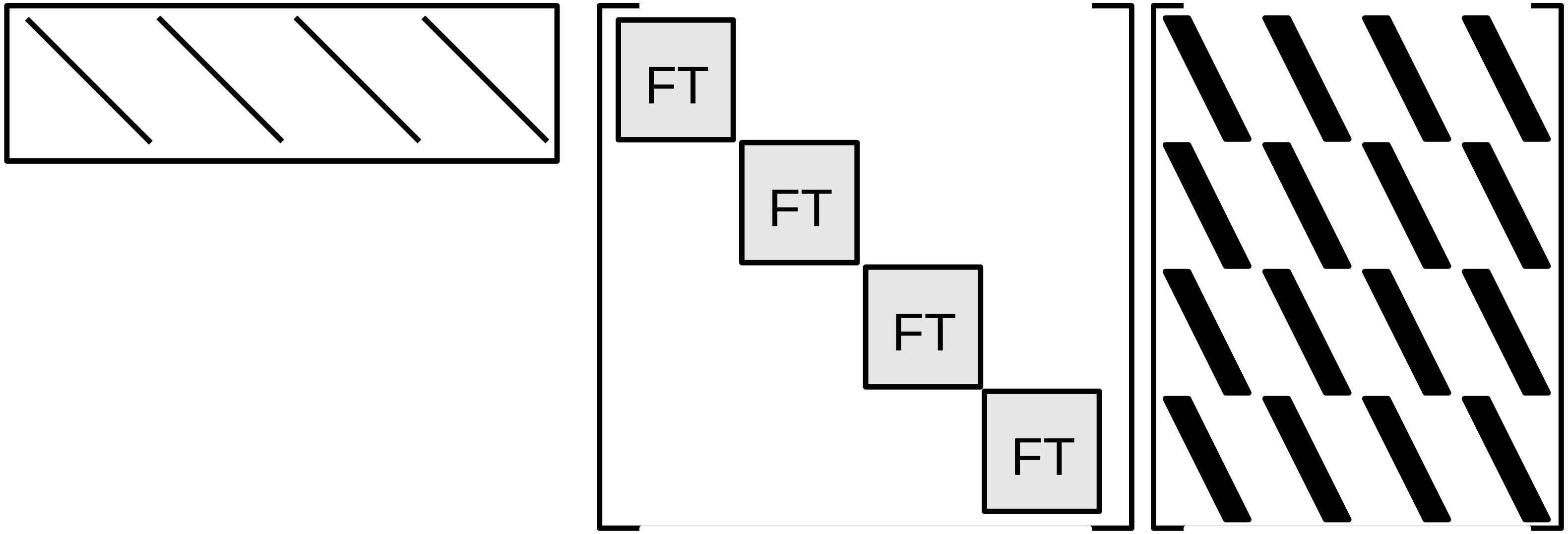


1 Fourier Transform +
$N_c \times N_{\text{kernel}}$ multiplications per missing sample

Channel-by-channel k-Space Kernels

GRAPPA, PARS…

$R =$



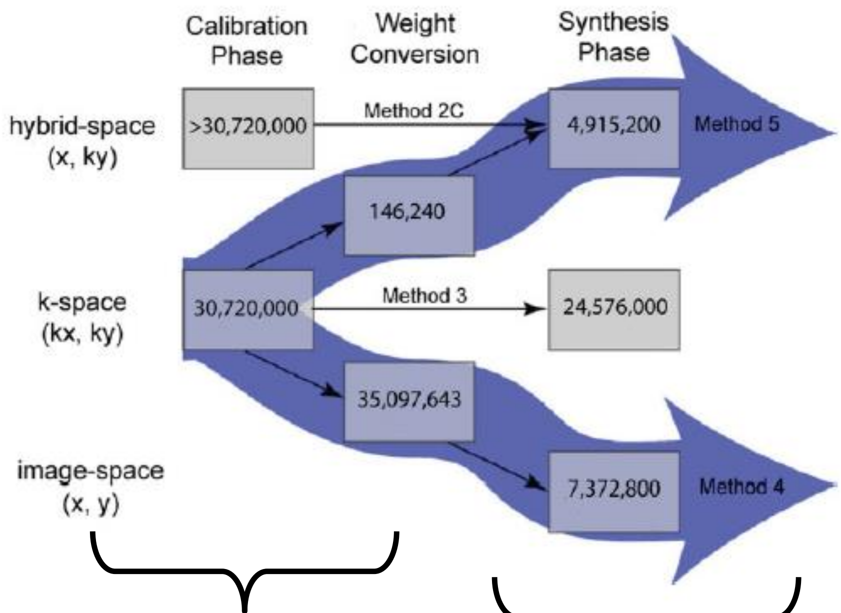
$N_{\mathrm{kernel}} {N_c}^2$ multiplications per missing sample +
N_c Fourier Transforms +
N_c multiplications per voxel

Mixing Reconstruction Components

Magnetic Resonance in Medicine 59:382–395 (2008)

Comparison of Reconstruction Accuracy and Efficiency Among Autocalibrating Data-Driven Parallel Imaging Methods

Anja C.S. Brau,[1*] Philip J. Beatty,[1] Stefan Skare,[2] and Roland Bammer[2]



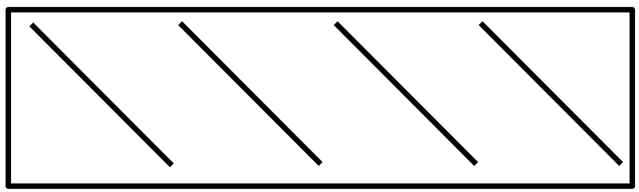
Generate R

Apply R

MRI Toolbox

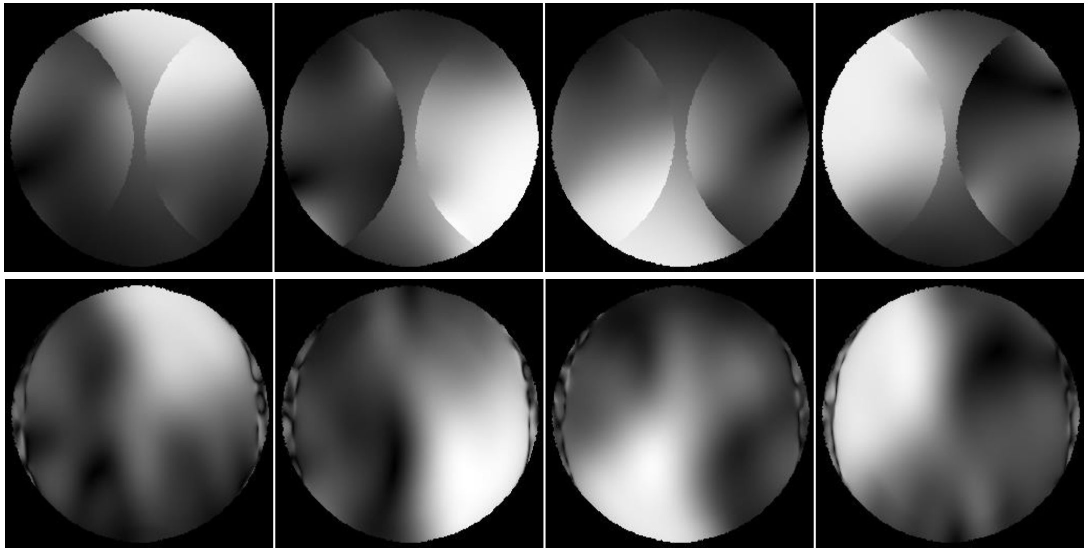
- Uniform sampling with image space synthesis
  - `calculate_sense_unmixing(…)`
  - `calculate_grappa_unmixing(…)`
  - `calculate_jer_unmixing(…)`

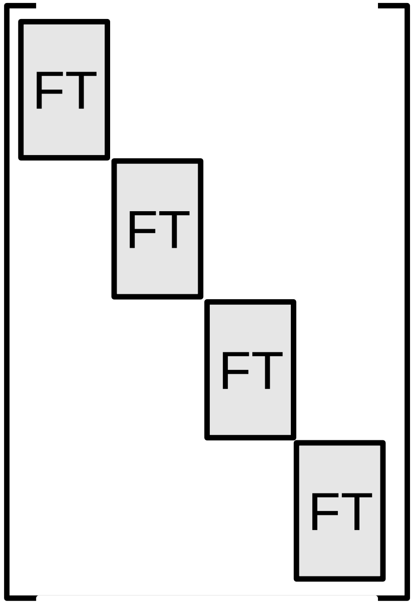
$R =$



SENSE unmixing

GRAPPA unmixing



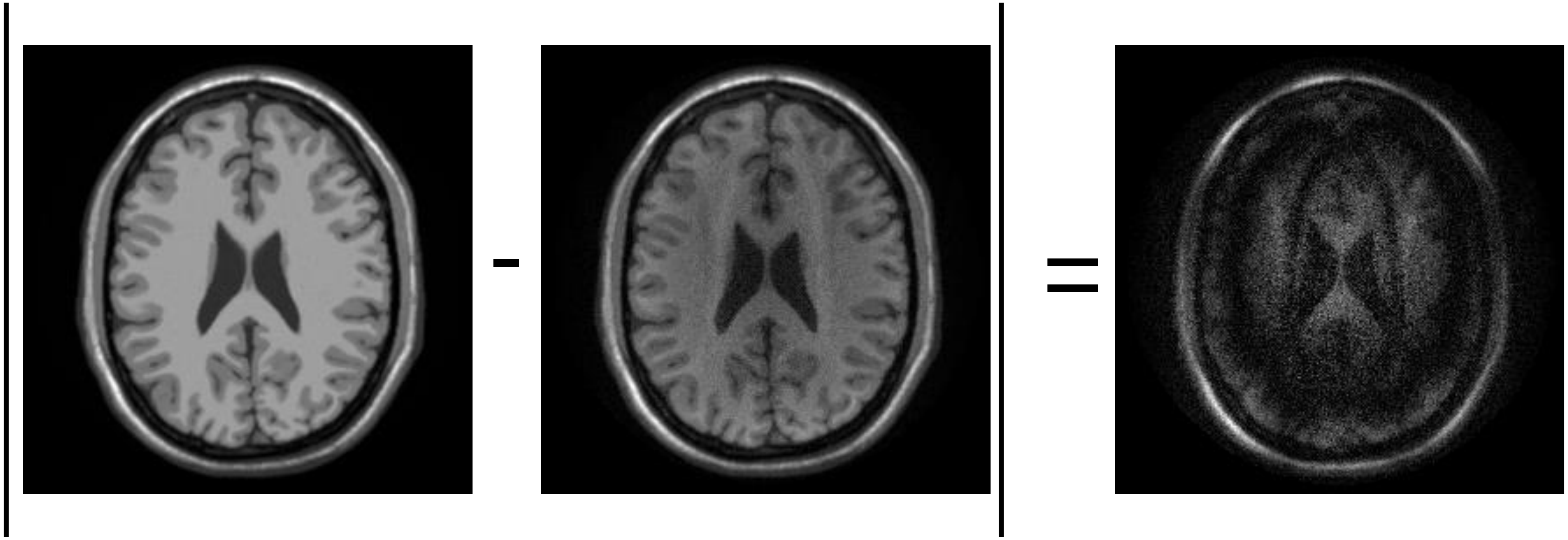


Application of local k-space kernels

Property	k-space	(x, ky, kz)-space	image space
Merge with channel combination	Hard	Hard	Easy
Non-uniform Cartesian sampling	Yes	Yes	No
Apply during data acquisition	Yes	Yes	No
Cost to transform kernels	None	Minimal	Moderate
Memory needed to store coefficients	$W_x W_y W_z N_c N_c$ 120KB*	$N_x W_y W_z N_c N_c$ 6MB*	$N_x N_y N_z N_c$ 1GB*
Application computation	$W_x W_y W_z N_c N_c$ per missing sample	$W_y W_z N_c N_c$ per missing sample	N_c per voxel

* Example based on 5x7x7 kernel for 256x256x256 image with 8 channels

Image Quality

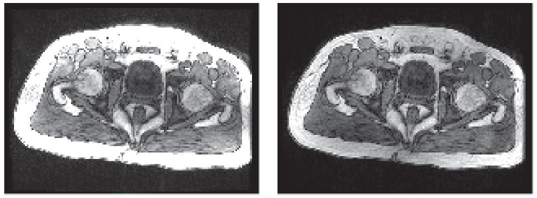


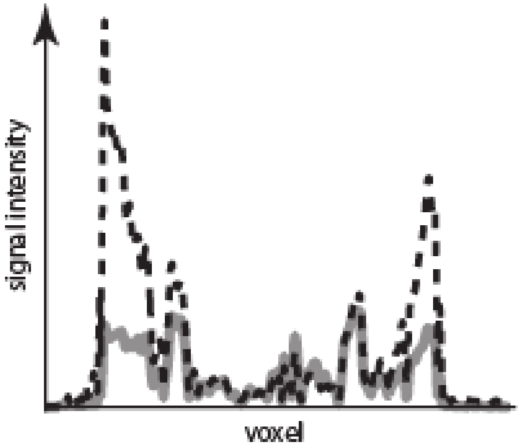
Shading?

Aliasing?

Noise?

Image Shading





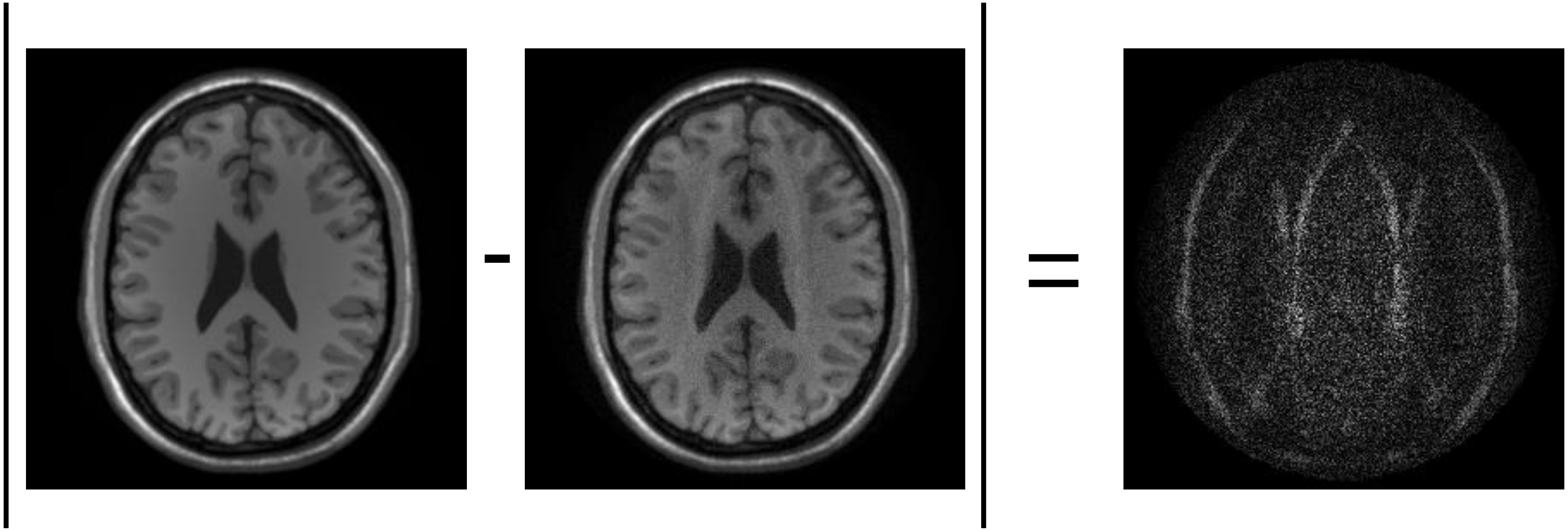
- Affects all multi-channel imaging (accelerated or not)
- Correction requires an absolute sensitivity reference to convert from *relative* coil sensitivities to *absolute* coil sensitivities.
  - e.g. calibration with uniformly sensitive reference coil or using uniform signal phantom/sequence.

Common Shading for Relative Coil Sensitivities

- Compare reconstruction methods without absolute reference
- Target profile: $\sqrt{\sum_{j=1}^{Nc} \left| s_j(x, y) \right|^2}$
- Same shading profile as a square-root sum-of-squares reconstruction.
- Take any relative channel combination maps, $c_j(x, y)$ and apply the following correction:

$$\hat{c}_j(x, y) = \frac{c_j(x, y)}{\sqrt{\sum_{j'=1}^{Nc} \left| c_{j'}(x, y) \right|^2}}$$

Image Quality



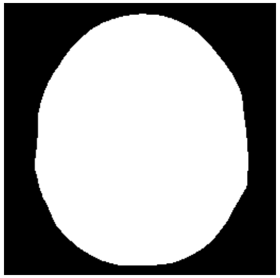
~~Shading?~~

Aliasing?

Noise?

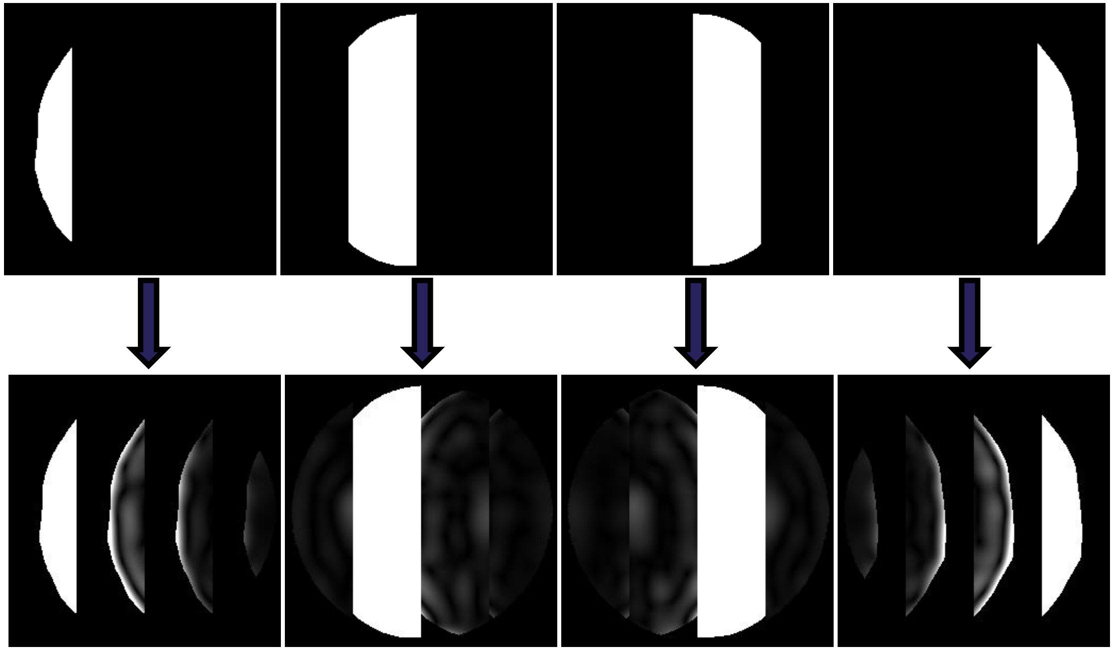
Aliasing Energy

pixel mask



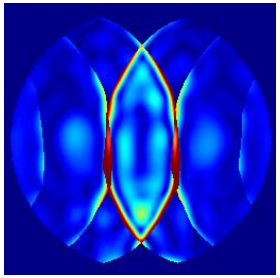
acceleration = 4

test signals



Signal spread

$\sqrt{\text{aliasing energy}}$



Coil Sensitivity Estimation

Magnetic Resonance in Medicine 43:682–690 (2000)

Adaptive Reconstruction of Phased Array MR Imagery

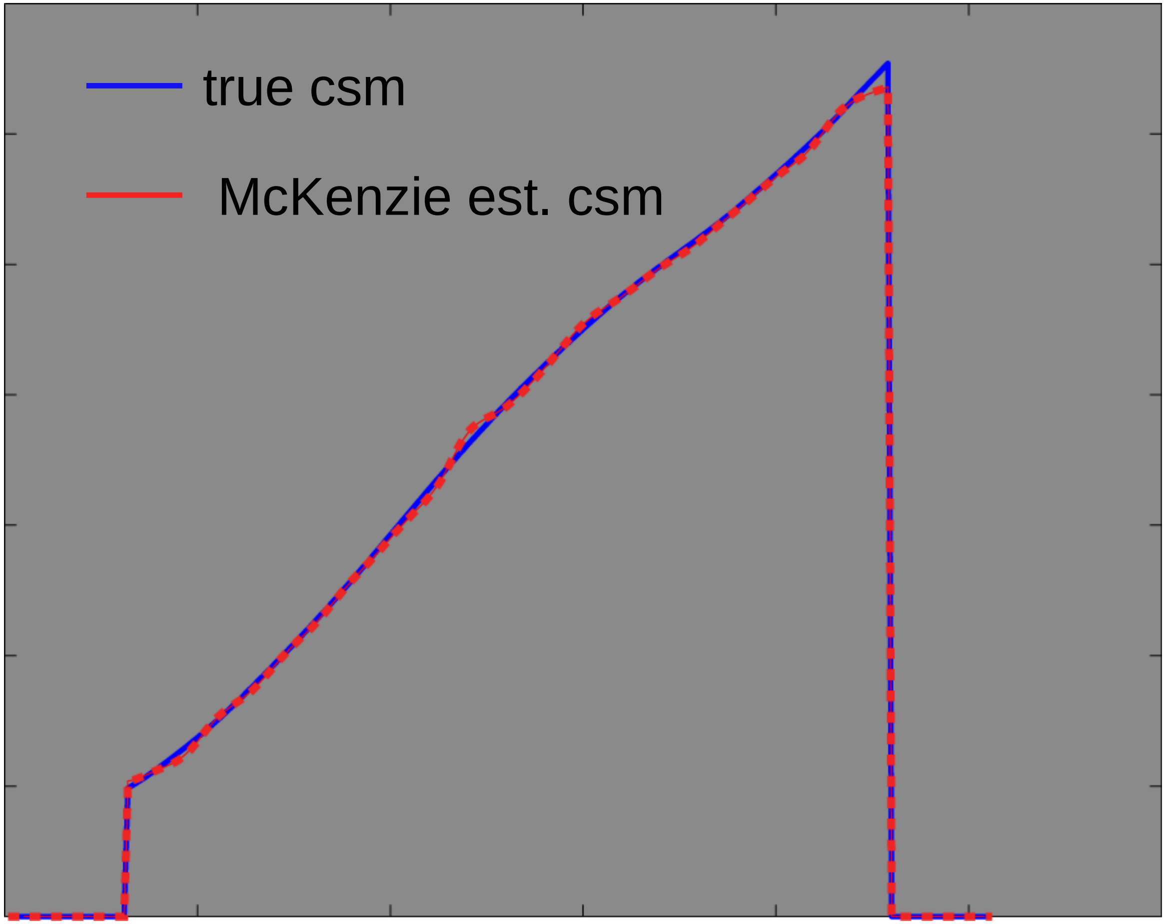
David O. Walsh,[1] Arthur F. Gmitro,[2*] and Michael W. Marcellin[3]

Magnetic Resonance in Medicine 47:529–538 (2002)
DOI 10.1002/mrm.10087

Self-Calibrating Parallel Imaging With Automatic Coil Sensitivity Extraction

Charles A. McKenzie,[1*] Ernest N. Yeh,[2] Michael A. Ohliger,[2]
Mark D. Price,[2] and Daniel K. Sodickson[1,2]

Coil Sensitivity Estimation



Local Kernel Calibration

Magnetic Resonance in Medicine 47:1202–1210 (2002)

Generalized Autocalibrating Partially Parallel Acquisitions (GRAPPA)

Mark A. Griswold,[1*] Peter M. Jakob,[1] Robin M. Heidemann,[1] Mathias Nittka,[2] Vladimir Jellus,[2] Jianmin Wang,[2] Berthold Kiefer,[2] and Axel Haase[1]

“Data Driven” $w = (D_s^H D_s)^{-1} D_s^H d_t$

Magnetic Resonance in Medicine 53:1383–1392 (2005)

3Parallel Magnetic Resonance Imaging with Adaptive Radius in *k*-Space (PARS): Constrained Image Reconstruction using *k*-Space Locality in Radiofrequency Coil Encoded Data

Ernest N. Yeh,[1,2] Charles A. McKenzie,[2] Michael A. Ohliger,[1,2] and Daniel K. Sodickson[1,2,3*]

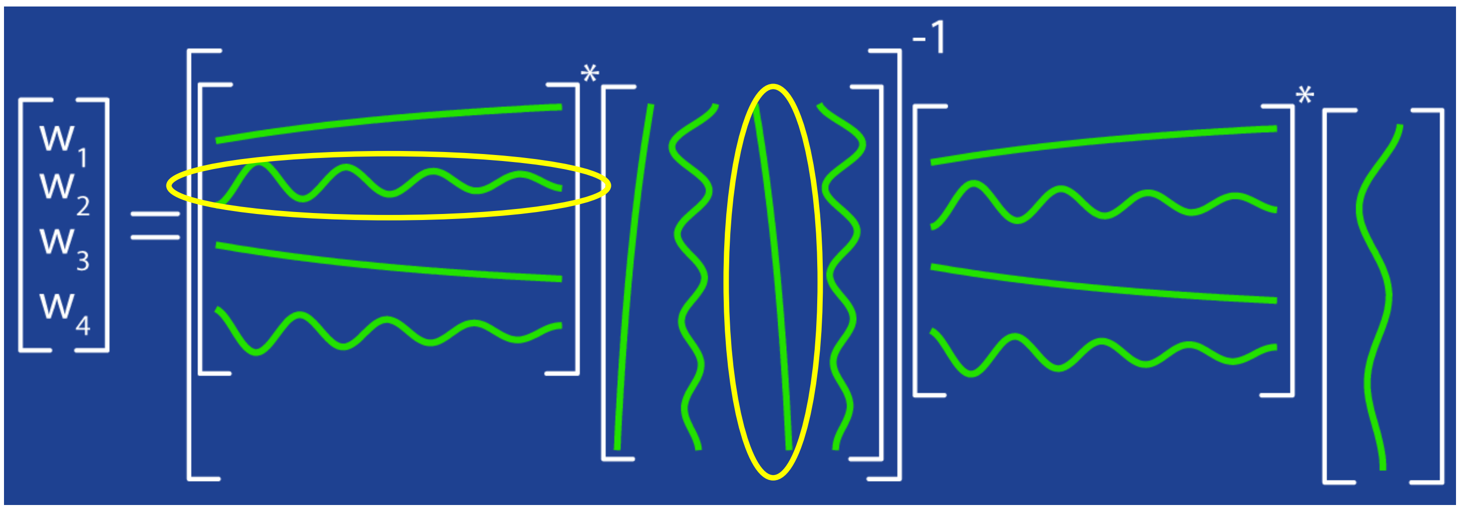
“Model Driven” $w = (E_s^H E_s)^{-1} E_s^H e_t$

Joint Encoding Relations

A Method for Autocalibrating 2-D Accelerated Volumetric Parallel Imaging with Clinically Practical Reconstruction Times

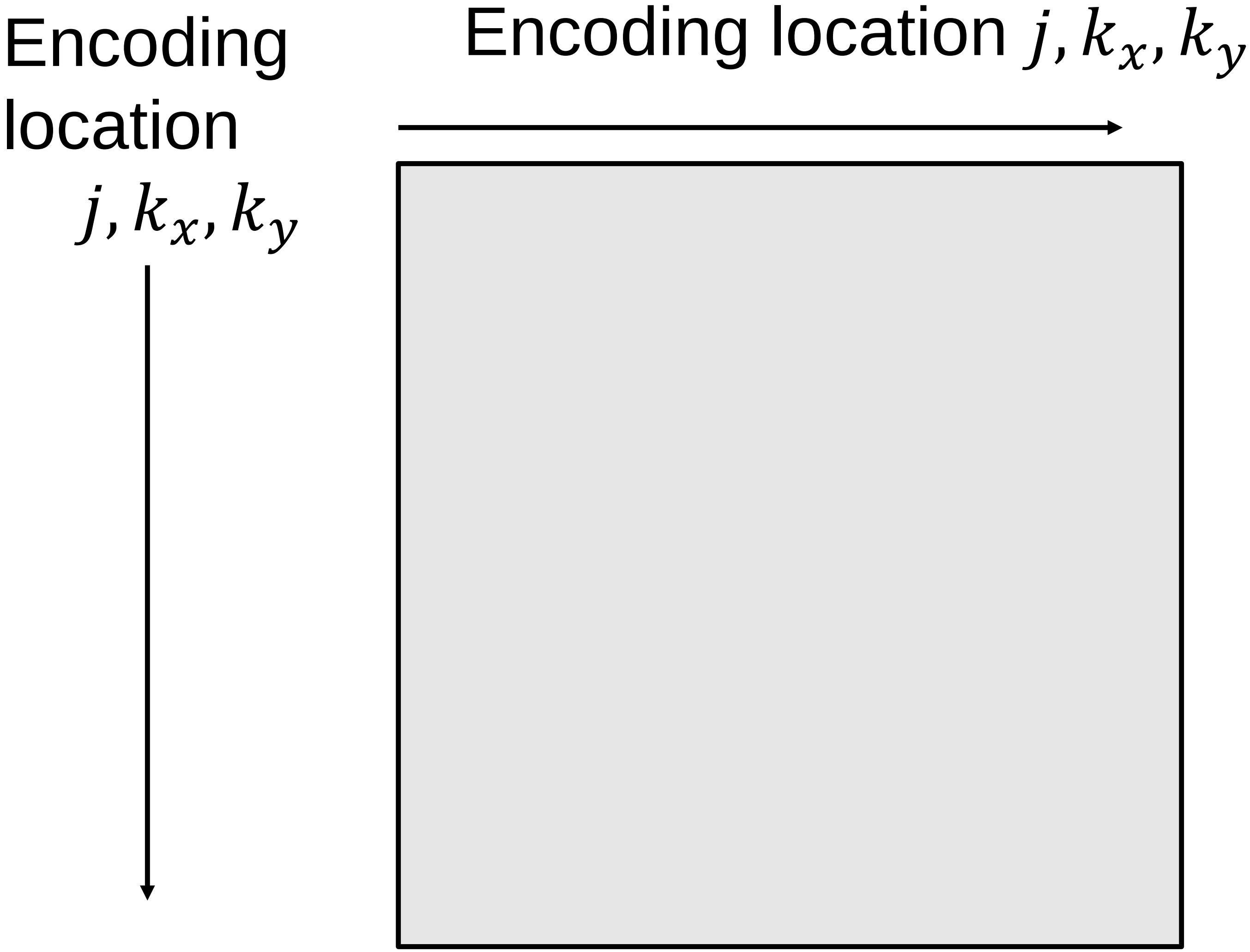
P. J. Beatty[1], A. C. Brau[1], S. Chang[2], S. M. Joshi[2], C. R. Michelich[2], E. Bayram[2], T. E. Nelson[3], R. J. Herfkens[3], and J. H. Brittain[4]

Proc. Intl. Soc. Mag. Reson. Med. 15 (2007) 1749



$$\langle e_1, e_2 \rangle$$

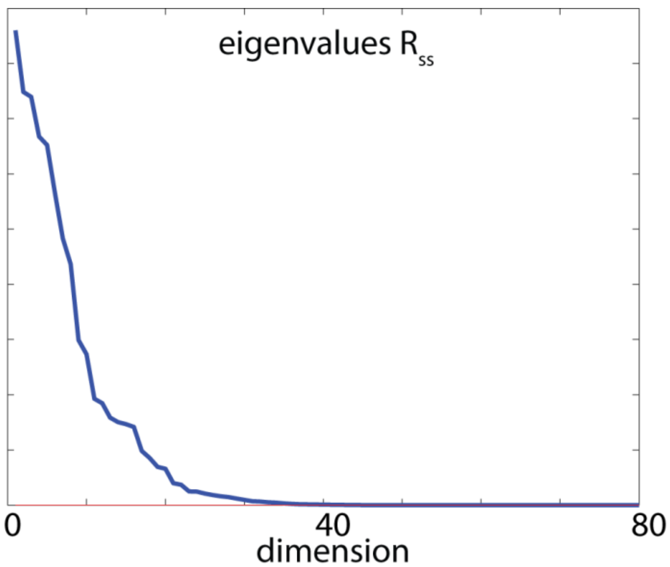
Joint Encoding Relations – Lookup Table

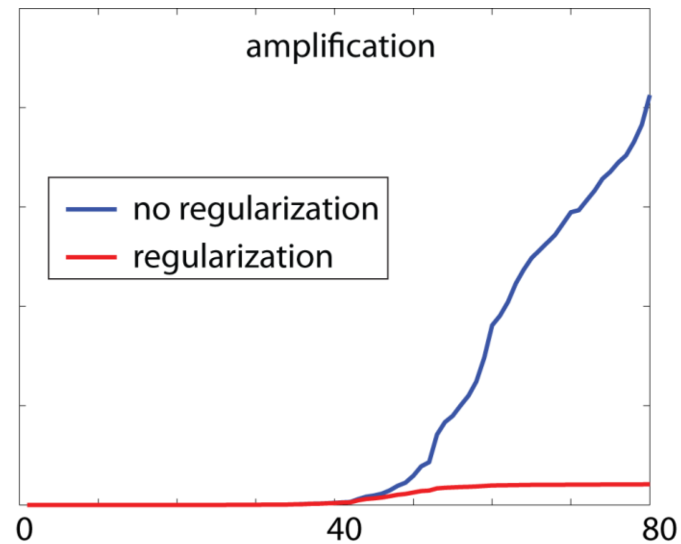


Joint Encoding Relations & Toolbox

- Image space GRAPPA
  - `ismrm_compute_jer_data_driven(…)`
  - `ismrm_calculate_jer_unmixing(…)`
- Image space PARS
  - `ismrm_compute_jer_model_driven(…)`
  - `ismrm_calculate_jer_unmixing(…)`

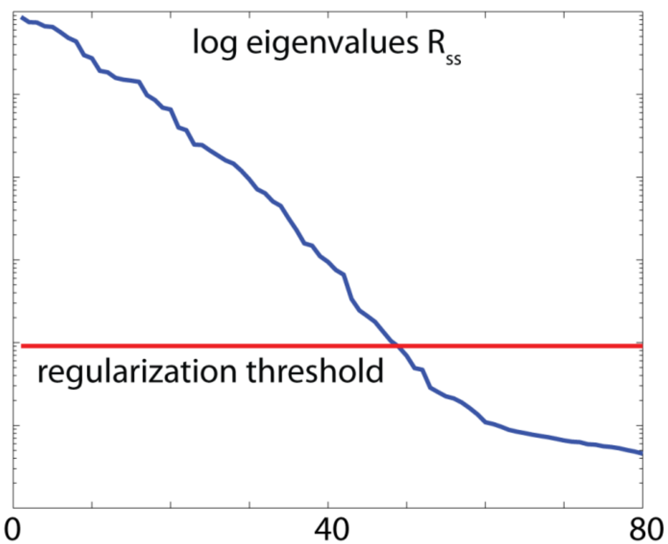
Tychonov Regularization

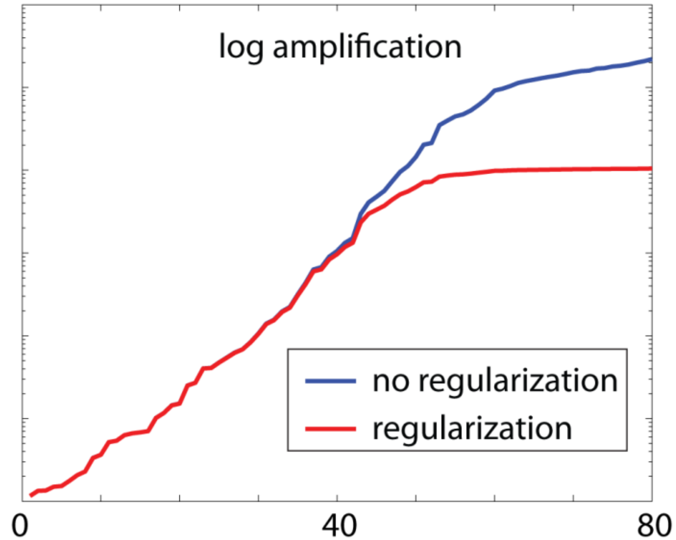


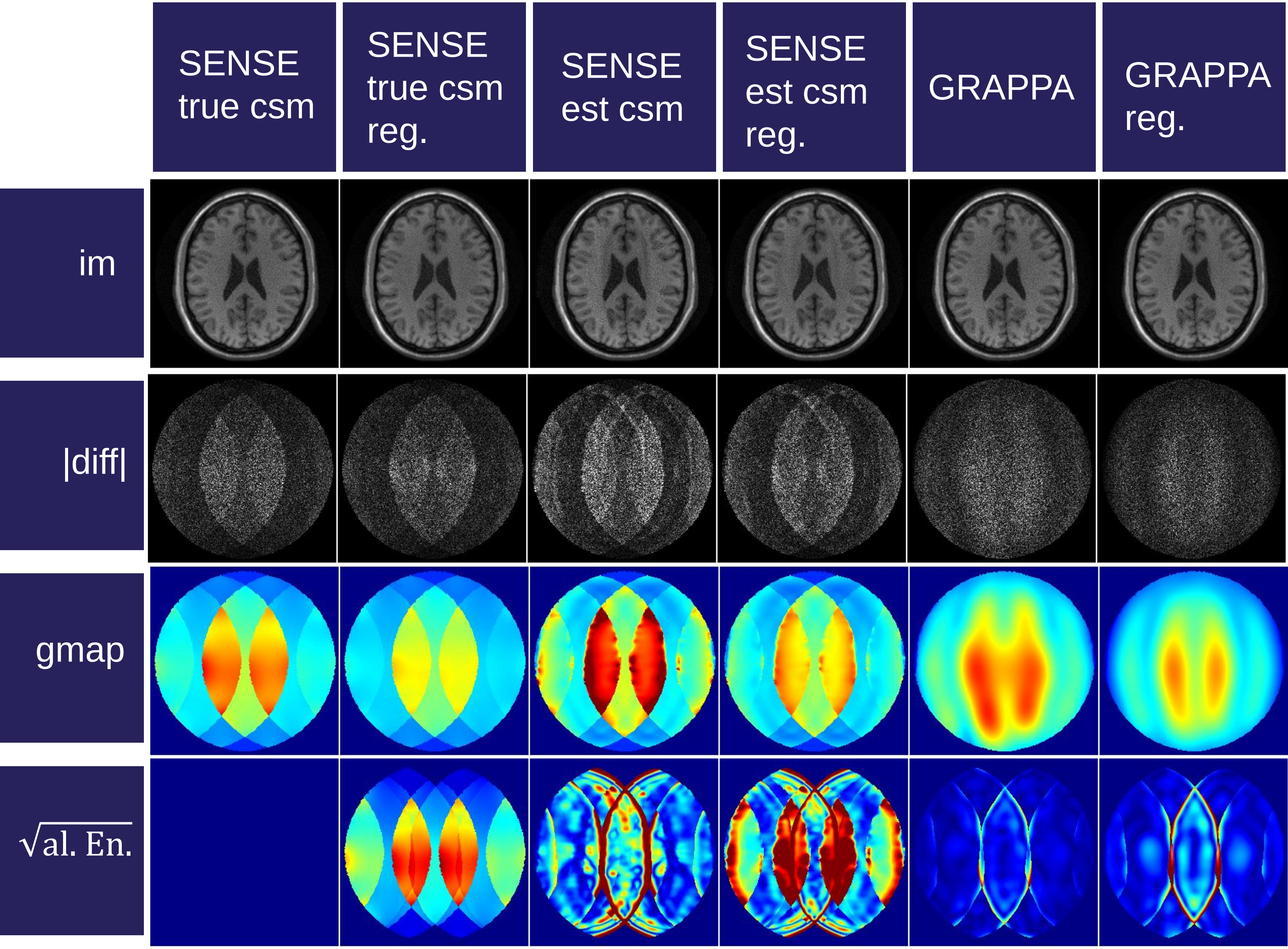


$$\frac{1}{\text{eigenvalue}}$$

$$\frac{1}{\lambda+\text{eigenvalue}}$$





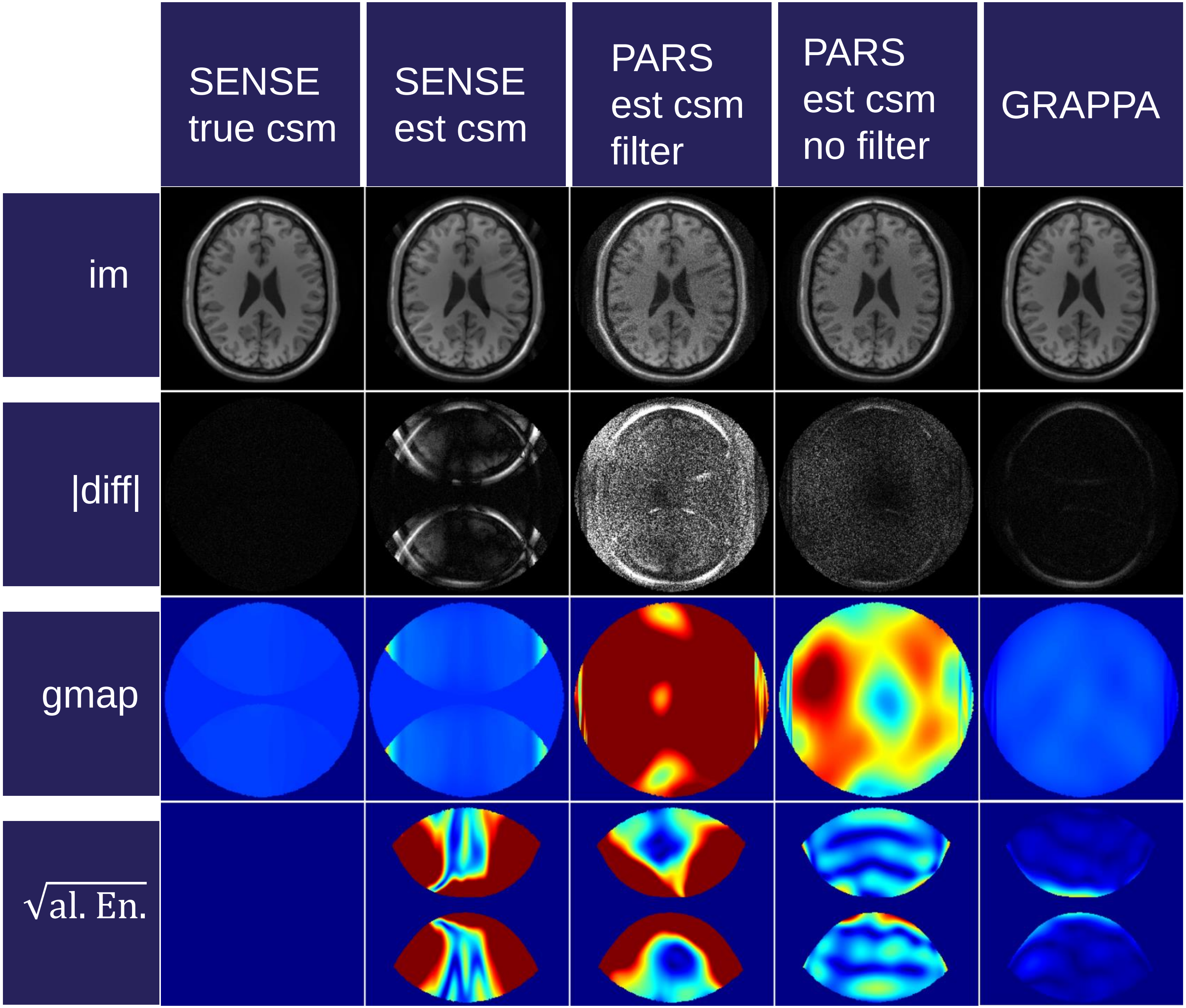

SENSE true csm
SENSE true csm reg.
SENSE est csm
SENSE est csm reg.
GRAPPA
GRAPPA reg.
im
|diff|
gmap
√al. En.

Minimal Calibration Data

- In some cases, collecting 32(+) lines of k-space for calibration is not feasible, or drastically reduces net acceleration. e.g. PROPELLER

- Challenging to estimate sensitivity maps:
  - Cal data is a low resolution image of the magnetization-weighted sensitivities: $\left[m(r)s_j(r)\right]*\mathrm{psf}(r)$
  - Even if sensitivities are low resolution, separating the sensitivity function is an approximation that cal lead to aliasing artifacts.

$$[m(r) * \mathrm{psf}(r)]s_j(r)$$


SENSE true csm
SENSE est csm
PARS est csm filter
PARS est csm no filter
GRAPPA
im
|diff|
gmap
$\sqrt{\text{al. En.}}$

Reduced FOV case

Magnetic Resonance in Medicine 52:1118–1126 (2004)

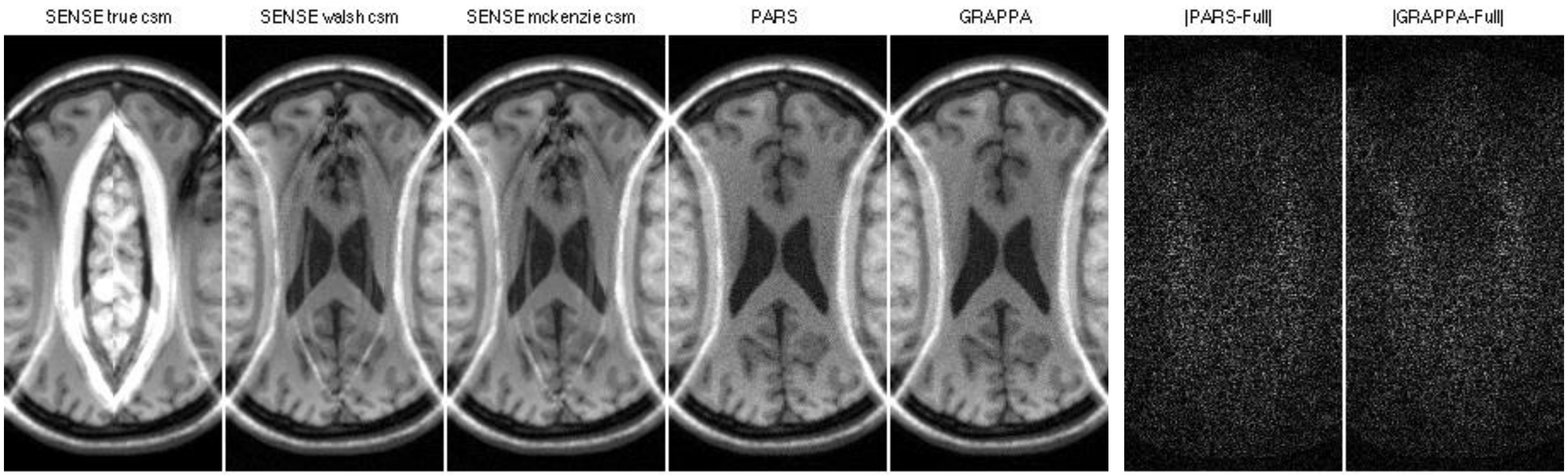
Field-of-View Limitations in Parallel Imaging

Mark A. Griswold,[1*] Stephan Kannengiesser,[2] Robin M. Heidemann,[1] Jianmin Wang,[2] and Peter M. Jakob[1]

Understanding the GRAPPA Paradox

P. J. Beatty[1,2], A. C. Brau[1]

Proc. Intl. Soc. Mag. Reson. Med. 14 (2006) 2467



- ismrm_demo_rFOV.m
- Low resolution unaliasing kernels; coil sensitivities with discontinuities
- Calibration approach impacts image quality

Summary

- Divide reconstruction components into separate components
  - Calibration approach impacts image quality
  - Data synthesis approach impacts reconstruction efficiency
  - Mix and match components to get desired behavior
- Tradeoffs between efficiency, artifacts and noise
  - Match operating point to target application
- Use tools to help separate shading, aliasing and noise degredation