Image Reconstruction – Parallel Imaging
Part I

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No conflicts of interest to disclose
Image Reconstruction Goal

- Instantaneous results
- Perfect signal fidelity with no artifacts
- No noise

load im1.mat
How far away are we?

Goal

You are here

Efficiency
- Computation operations
- Computation time
- Memory usage

Artifacts
- Image shading
- Aliasing energy

Noise
- G-factor maps
- Noise amplification maps
- SNR-scaled reconstruction
Concept of Sensitivity Encoding
Encoding Model

\[ d_j(k_x, k_y) = \iiint s_j(x, y)e^{i2\pi(k_xx+k_yy)}m(x, y)\,dx\,dy \]

\[ d = Em + \text{noise} \]
Reconstruction

\[ \hat{m} = (E^H E)^{-1} E^H d \]

\[ \hat{m} = Rd \]
Reconstruction Matrix Design

\[ \hat{m} = R \cdot E \cdot m \]

Linear combination of acquired encoding functions to give desired encoding function
Reconstruction Components

1. Estimate $E$
   - $s_j(x,y)$ Estimate coil sensitivities

2. Generate $R$
   - One possibility: $R = \text{pinv}(E)$

3. Apply $R$
   - $\hat{m} = Rd$
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Image Quality
Reconstruction Efficiency
Reconstruction Components: Noniterative Methods

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Reconstruction Components: Iterative Methods

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   - $\hat{m} = Rd$

Calibration

Fused in iterative loop
1. Estimate $E$
   
   - $s_j(x, y)$ Estimate coil sensitivities

2. Generate $R$
   
   - One possibility: $R = \text{pinv}(E)$

   "unfolding matrix"

   $$U = (S^H \Psi^{-1} S)^{-1} S^H \Psi^{-1}$$
3. **Apply** \( R \)
   - \( \hat{m} = Rd \)

\[ R = \text{FFT} \]

**mag**

**phase**

\( N_c \) Fourier Transforms + \( N_c \) multiplications per voxel

“unmixing images”
Local k-Space Kernels

- Enable non-iterative reconstruction of non-uniform sampling patterns.

SMASH
PARS
AUTO-SMASH
VD-AUTO-SMASH
GRAPPA

…and more
Composite Channel Local k-Space Kernels

SMASH, AUTO-SMASH, VD-AUTO-SMASH,…

\[ R = \begin{bmatrix} \text{FT} \\ \vdots \end{bmatrix} \begin{bmatrix} \text{N} \\ \text{c} \end{bmatrix} \times N_{\text{kernel}} \text{ multiplications per missing sample} \]

1 Fourier Transform +

Apply kernels
Channel-by-channel k-Space Kernels

GRAPPA, PARS…

\[ R = \begin{bmatrix} \text{FT} \\ \text{FT} \\ \text{FT} \end{bmatrix} \]

\[ N_{\text{kernel}} \cdot N_c^2 \text{ multiplications per missing sample} + \]
\[ N_c \text{ Fourier Transforms} + \]
\[ N_c \text{ multiplications per voxel} \]
Mixing Reconstruction Components

Comparison of Reconstruction Accuracy and Efficiency Among Autocalibrating Data-Driven Parallel Imaging Methods

Anja C.S. Brau, Philip J. Beatty, Stefan Skare, and Roland Bammer

Generate R

Apply R
MRI Toolbox

- Uniform sampling with image space synthesis
  - `calculate_sense_unmixing(...)`
  - `calculate_grappa_unmixing(...)`
  - `calculate_jer_unmixing(...)`

\[ R = \begin{bmatrix}
  & & & \\
  & & & \\
  & \text{FT} & & \\
  & \text{FT} & & \\
\end{bmatrix} \]

SENSE unmixing

GRAPPA unmixing
## Application of local k-space kernels

<table>
<thead>
<tr>
<th>Property</th>
<th>k-space</th>
<th>(x, ky, kz)-space</th>
<th>image space</th>
</tr>
</thead>
<tbody>
<tr>
<td>Merge with channel combination</td>
<td>Hard</td>
<td>Hard</td>
<td>Easy</td>
</tr>
<tr>
<td>Non-uniform Cartesian sampling</td>
<td>Yes</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td>Apply during data acquisition</td>
<td>Yes</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td>Cost to transform kernels</td>
<td>None</td>
<td>Minimal</td>
<td>Moderate</td>
</tr>
<tr>
<td>Memory needed to store coefficients</td>
<td>$W_x W_y W_z N_c N_c$ 120KB*</td>
<td>$N_x W_y W_z N_c N_c$ 6MB*</td>
<td>$N_x N_y N_z N_c$ 1GB*</td>
</tr>
<tr>
<td>Application computation</td>
<td>$W_x W_y W_z N_c N_c$ per missing sample</td>
<td>$W_y W_z N_c N_c$ per missing sample</td>
<td>$N_c$ per voxel</td>
</tr>
</tbody>
</table>

* Example based on 5x7x7 kernel for 256x256x256 image with 8 channels
Image Quality

Shading?  Aliasing?  Noise?
Image Shading

- Affects all multi-channel imaging (accelerated or not)

- Correction requires an absolute sensitivity reference to convert from relative coil sensitivities to absolute coil sensitivities.
  - e.g. calibration with uniformly sensitive reference coil or using uniform signal phantom/sequence.
Common Shading for Relative Coil Sensitivities

- Compare reconstruction methods without absolute reference.

- Target profile: \( \sqrt{\sum_{j=1}^{N_{c}} |s_j(x, y)|^2} \)

- Same shading profile as a square-root sum-of-squares reconstruction.

- Take any relative channel combination maps, \( c_j(x, y) \) and apply the following correction:

\[
\hat{c}_j(x, y) = \frac{c_j(x, y)}{\sqrt{\sum_{j'=1}^{N_{c}} |c_{j'}(x, y)|^2}}
\]
Image Quality

Shading?

Aliasing?

Noise?
Aliasing Energy

pixel mask

acceleration = 4

test signals

Signal spread

\sqrt{\text{aliasing energy}}
Coil Sensitivity Estimation

Adaptive Reconstruction of Phased Array MR Imagery
David O. Walsh,¹ Arthur F. Gmitro,²* and Michael W. Marcellin³

Self-Calibrating Parallel Imaging With Automatic Coil Sensitivity Extraction
Charles A. McKenzie,¹* Ernest N. Yeh,² Michael A. Ohliger,² Mark D. Price,² and Daniel K. Sodickson¹,²
Coil Sensitivity Estimation

true csm

McKenzie est. csm
Local Kernel Calibration

“Data Driven” \( w = (D_S^H D_S)^{-1} D_S^H d_t \)

“Model Driven” \( w = (E_S^H E_S)^{-1} E_S^H e_t \)
Joint Encoding Relations

\[ \langle e_1, e_2 \rangle \]
Joint Encoding Relations – Lookup Table

Encoding location $j, k_x, k_y$

Encoding location $j, k_x, k_y$
Joint Encoding Relations & Toolbox

- **Image space GRAPPA**
  - ismrn_compute_jer_data_driven(...)
  - ismrn_calculate_jer_unmixing(...)

- **Image space PARS**
  - ismrn_compute_jer_model_driven(...)
  - ismrn_calculate_jer_unmixing(...)
Tychonov Regularization

$\frac{1}{\lambda + \text{eigenvalue}}$

$\frac{1}{\text{eigenvalue}}$
Minimal Calibration Data

- In some cases, collecting 32(+) lines of k-space for calibration is not feasible, or drastically reduces net acceleration. e.g. PROPELLER

- Challenging to estimate sensitivity maps:
  - Cal data is a low resolution image of the magnetization-weighted sensitivities: \[ m(r)s_j(r) \ast psf(r) \]
  - Even if sensitivities are low resolution, separating the sensitivity function is an approximation that can lead to aliasing artifacts. \[ m(r) \ast psf(r)s_j(r) \]
<table>
<thead>
<tr>
<th></th>
<th>SENSE true csm</th>
<th>SENSE est csm</th>
<th>PARS est csm filter</th>
<th>PARS est csm no filter</th>
<th>GRAPPA</th>
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- SENSE: Single-Excitation Sensitivity Encoding
- PARS: Partially-Anticipated Reconciliation
- GRAPPA: Generalized Autocalibrating Partially Parallel Acquisitions
- true csm: True Constrained Sensitivity Map
- est csm: Estimated Constrained Sensitivity Map
- filter: Filtered reconstruction
- no filter: Unfiltered reconstruction

- diff: Image difference
- gmap: Gradient map
- √al. En.: Square root of absolute value of the estimation

*Note: The images show the results of different reconstruction techniques applied to MR images, comparing the effects of true and estimated sensitivity maps with and without filtering.*
Reduced FOV case

- ismrn_demo_rFOV.m

- Low resolution unaliasing kernels; coil sensitivities with discontinuities

- Calibration approach impacts image quality
Summary

- Divide reconstruction components into separate components
  - Calibration approach impacts image quality
  - Data synthesis approach impacts reconstruction efficiency
  - Mix and match components to get desired behavior

- Tradeoffs between efficiency, artifacts and noise
  - Match operating point to target application

- Use tools to help separate shading, aliasing and noise degradation